

# Dining Philosophers: A Synchronization Problem

CS347 Operating Systems

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*When you go to sleep make sure there is someone to wake you up.*

*(Prof. Mythili Vutukuru)*

# Outline

- 1 Problem Formulation
  - The Setup
  - The Problem
- 2 Semaphores – Focusing on Forks
  - Introduction
  - Incorrect Solution
  - Correct Solution
- 3 Condition Variables – Focusing on Philosophers
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# Problem Formulation

## The Setup

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  - ▶ a huge bowl of spaghetti in the centre of table.
- $P_i$  has  $p_i$  in their front and  $f_i, f_{(i+1)\%N}$  to their right and left respectively.

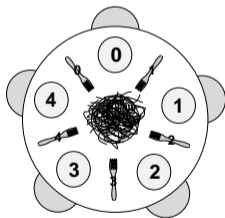


Figure: Example<sup>1</sup> when  $N = 5$

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- A philosopher can start eating only after picking up the forks on their left and right.
- Till they start eating, they will be 'thinking'.
- Generic Behaviour of a philosopher:

```
while (True){  
    think()  
    pick_up_forks()  
    eat()  
    put_down_forks()  
}
```

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- A complicated problem as a thread can possibly context switch anytime during its execution

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- The right and left philosopher for  $i^{\text{th}}$  philosopher are given by:

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- A philosopher  $P_0$  successfully completes/finishes when it goes back to thinking.
- A scheduled thread is active when it has run at least once.

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  - ▶ When a thread calls  $\text{down}(s)$ , the counter is decremented and the thread is blocked if the counter value becomes negative.
  - ▶ When a thread calls  $\text{up}(s)$ , the counter is incremented and any one of blocked threads is woken up ('ready to run' again).

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- Pseudocode:

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function pick_up_forks(philosopher i){  
    down(s_{right_f(i)})  
    down(s_{left_f(i)})  
}
```

```
function put_down_forks(philosopher i){  
    up(s_{left_f(i)})  
    up(s_{right_f(i)})  
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Example – Deadlock

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- $P_1$  will get the right fork  $f_1$  by calling  $\text{down}(s_{\{\text{right\_f}(1)\}}) = \text{down}(s_1)$  which will make  $s_1 = 0$ . Now, suppose  $P_1$  gets context switched then assuming  $P_2$  begins,

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- $P_2$  will get the right fork  $f_2$  by calling  $\text{down}(s_{\{\text{right\_f}(2)\}}) = \text{down}(s_2)$  which will make  $s_2 = 0$ . Now, every  $P_i$  will have a single fork  $f_i$ .

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- Now, if any thread  $P_i$  executes  $\text{down}(s_{\{\text{left\_f}(i)\}})$ , then for  $f_{\text{left}(i)}$ ,  $s_{\text{left}(i)} = -1$ . Hence, that thread will be sent to sleep.

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- Each thread will try to access their 'left fork' and will be sent to eternal sleep. A *deadlock*!



# Semaphores

Incorrect Solution – Why Deadlock?

Proof.

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- Now, if any thread  $P_i$  gets scheduled and executes  $\text{down}(s_{\text{left\_f}(i)})$ , then for that fork's semaphore  $s_{\text{left}(i)} = -1$ . Hence, that thread will be sent to sleep.

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- Similarly, each thread will try to access their 'left fork' and will be sent to sleep.

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- Similarly, each thread will try to access their 'left fork' and will be sent to sleep.
- To awake them, some thread must give signal which is not possible as all threads are sleeping. A deadlock!

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- Similarly, each thread will try to access their 'left fork' and will be sent to sleep.
- To awake them, some thread must give signal which is not possible as all threads are sleeping. A deadlock!
- Note that the above case is possible for any scheduling of threads as we haven't made any assumption on scheduling in the proof.



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- In addition to the  $N$  semaphore variables, one for each fork  $s_i = 1, i \in [N]$ ,
- create another semaphore variable called  $max = N - 1$ , denoting the maximum number of philosophers allowed to eat at any instant.

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- create another semaphore variable called  $max = N - 1$ , denoting the maximum number of philosophers allowed to eat at any instant.
- Revised pseudocode:

```
function pick_up_forks(philosopher i){
    down(max)
    down(s_{right_f(i)})
    down(s_{left_f(i)})
}
```

```
function put_down_forks(philosopher i){
    up(s_{left_f(i)})
    up(s_{right_f(i)})
    up(max)
}
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- $P_0$  will get both it's 'right and left fork' ( $f_0$  and  $f_1$ ) and  $P_0$  will start eating.

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- $P_0$  will get both it's 'right and left fork' ( $f_0$  and  $f_1$ ) and  $P_0$  will start eating.
- Now, one of the two things can happen:
  - ▶  $P_0$  successfully completes everything
  - ▶  $P_0$  gets context switched out before it can put down its both forks.



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- In the first case,  $P_1$  will get both it's 'right and left fork' and it will start eating using  $f_1$  and  $f_2$ , whereas in the second case  $P_1$  will only be able to get one fork ( $f_2$ ) and will go to sleep.

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- $P_0$  will complete eating and thus its job at sometime due to assumption that eat has to terminate. So, forks  $f_0$  and  $f_1$  will be available later for other threads.

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- $P_0$  will complete eating and thus its job at sometime due to assumption that eat has to terminate. So, forks  $f_0$  and  $f_1$  will be available later for other threads.
- Even in the 2<sup>nd</sup> case after completion of  $P_0$ ,  $P_1$  will wake up and start eating using  $f_1$  and  $f_2$ .

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- In the first case,  $P_1$  will get both its 'right and left fork' and it will start eating using  $f_1$  and  $f_2$ , whereas in the second case  $P_1$  will only be able to get one fork ( $f_2$ ) and will go to sleep.
- $P_0$  will complete eating and thus its job at sometime due to assumption that eat has to terminate. So, forks  $f_0$  and  $f_1$  will be available later for other threads.
- Even in the 2<sup>nd</sup> case after completion of  $P_0$ ,  $P_1$  will wake up and start eating using  $f_1$  and  $f_2$ .
- Hence,  $P_2$  will get a fork  $f_0$  and will go to sleep waiting for  $P_1$  to complete.

# Semaphores – Correct Solution

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- Hence,  $P_2$  will get a fork  $f_0$  and will go to sleep waiting for  $P_1$  to complete.
- Once  $P_1$  completes eventually,  $P_2$  will be woken up and it will also get completed.

# Semaphores – Correct Solution

## Example 2

Example

# Semaphores – Correct Solution

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# Semaphores – Correct Solution

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- Say for  $N = 3$  case, the schedule is  $P_0, P_1, P_2$
- If any thread is able to pick up both forks, it will eventually finish and the analysis will be similar to Example 1.

# Semaphores – Correct Solution

## Example 2

Example (continued)

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- So let's try the worst case, when each thread gets context switched just after picking up right fork, now the *max* semaphore comes into picture.

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- So let's try the worst case, when each thread gets context switched just after picking up right fork, now the *max* semaphore comes into picture.
- $P_0$  and  $P_1$  will get their right fork  $f_0, f_1$  respectively.
- Only two of three threads can be active at a time. As both  $P_0$  and  $P_1$  are yet not completed, When  $P_2$  tries to pick up forks, it is sent to sleep as *max* becomes  $-1$ .

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- So,  $P_0$  and  $P_1$ , will be able to pick up atleast one fork i.e. the uncommon forks.
- Also, one of them will surely pick up the common fork between them  $f_1$ . So that thread will start eating.

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- So let's try the worst case, when each thread gets context switched just after picking up right fork, now the *max* semaphore comes into picture.
- $P_0$  and  $P_1$  will get their right fork  $f_0$ ,  $f_1$  respectively.
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- Eventually, it will finish and now the other thread will pick up  $f_1$  and start eating .



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- Eventually, it will finish and now the other thread will pick up  $f_1$  and start eating .
- Now,  $P_2$  will be woken up and once the older thread finishes,  $P_2$  will start eating.

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- Due to *max*, at most  $N - 1$  threads have begun but there are  $N$  forks. So, by pigeonhole principle one thread (say  $P_i$ ) will get two forks which will be done by `pick_up_forks`.

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- Semaphore variables, ensure that multiple threads can't access same fork at a time.



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- So eventually  $P_i$  will finish and we will be left with one less thread.
- If one thread was able to finish when a total of  $k$  threads were active, then one thread can definitely finish when a total of  $k - 1$  threads were active as we can always add a thread which does nothing.

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- If one thread was able to finish when a total of  $k$  threads were active, then one thread can definitely finish when a total of  $k - 1$  threads were active as we can always add a thread which does nothing.
- So, we have recursively shown that all threads will finish.



# Condition Variables

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  - ▶ When a thread calls `wait(cv)`, it is added to a list of waiting threads for `cv` and is blocked. This list is maintained for every condition variable.
  - ▶ When a thread calls `signal(cv)`, any one of blocked threads is woken up ('ready to run' again). There is no immediate context switch, it will be scheduled later.

# Condition Variables – Focusing on Philosophers

Incorrect Solution

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- Pseudocode:

```
function pick_up_forks(philosopher i){
    while (s_{right_p(i)} = E OR s_{left_p(i)} = E)
        wait(c_i)
    s_i = E
}
```

```
function put_down_forks(philosopher i){
    s_i = T
    if (s_{right_p(right_p(i))} = T)
        signal(c_{right_p(i)})
    if (s_{left_p(left_p(i))} = T)
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# Condition Variables – Incorrect Solution

## Example 1 – Deadlock

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- Now when  $P_0$  completes eating it will execute `put_down_forks`, which changes its state back to thinking and sends the signal to wake up  $P_1$  and  $P_2$  using  $c_1$  and  $c_2$  respectively.

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- Now, when  $P_1$  and  $P_2$  will come back they will execute the wait statement ignorant of the fact that  $P_0$  is already completed
- So both  $P_1$  and  $P_2$ , will go to eternal sleep, called a *missed wakeup* problem and a deadlock!

# Condition Variables – Incorrect Solution

## Example 2 – Race Condition

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- For  $P_1$ , both its 'right and left neighbour' ( $P_0$  and  $P_2$ ) are thinking. So,  $P_1$  exits while loop and changes its state to eating.

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- Now even if  $P_2$  is scheduled next, it will remain in while loop and go to sleep as  $P_1$  is eating. So  $P_0$  comes back and changes its state to eating.
- The above will imply two neighbouring philosophers are eating simultaneously by using a common fork which should not happen, a *race condition*!

# Condition Variables

Correct Solution

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- Pseudocode:

```
function pick_up_forks(philosopher i){
    lock(mutex)
    while (s_{right_p(i)} = E OR s_{left_p(i)} = E)
        wait(c_i, mutex)
    s_i = E
    unlock(mutex)
}
```

```
function put_down_forks(philosopher i){
    lock(mutex)
    s_i = T
    if (s_{right_p(right_p(i))} = T)
        signal(c_{right_p(i)})
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        signal(c_{left_p(i)})
    unlock(mutex)
}
```

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- Suppose,  $P_1$  is context switched just before it can go to sleep. Then as  $P_1$  has not yet released the lock,  $P_0, P_2$  will keep waiting for `put_down_forks` and `pick_up_forks` respectively.

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- The lock will be released only when  $P_1$  comes back and executes `wait`. The lock is released only after ensuring that list of waiting processes contains  $P_1$ . Then  $P_1$  goes to sleep.

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- The lock will be released only when  $P_1$  comes back and executes `wait`. The lock is released only after ensuring that list of waiting processes contains  $P_1$ . Then  $P_1$  goes to sleep.
- If  $P_2$  is scheduled earlier than  $P_0$ , then it will go to sleep eventually (directly or like in  $P_1$ ).

# Condition Variables – Correct Solution

## Example 1

Example (continued)

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### Example (continued)

- When  $P_0$  completes eating it will execute `put_down_forks` and acquire the lock then change its state back to thinking and signal  $P_1$  and  $P_2$  using  $c_1$  and  $c_2$  respectively.



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- Now, whoever among  $P_1$  and  $P_2$  will come back first (say  $P_1$ ) will reacquire lock and will release the lock only after changing its state and then it can start eating.

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- Now, whoever among  $P_1$  and  $P_2$  will come back first (say  $P_1$ ) will reacquire lock and will release the lock only after changing its state and then it can start eating.
- Even if there are more random context switches  $P_1$  will start eating first as  $P_2$  will have to wait till the  $P_1$  state changes back to thinking. So,  $P_1$  will eventually finish.

# Condition Variables – Correct Solution

## Example 1

### Example (continued)

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- Now, whoever among  $P_1$  and  $P_2$  will come back first (say  $P_1$ ) will reacquire lock and will release the lock only after changing its state and then it can start eating.
- Even if there are more random context switches  $P_1$  will start eating first as  $P_2$  will have to wait till the  $P_1$  state changes back to thinking. So,  $P_1$  will eventually finish.
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# Condition Variables – Correct Solution

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- When  $P_0$  comes back, it will change its state and release the lock and signal any one of neighbours (say  $P_1$ ) to wake up. Also,  $P_0$  can now start eating.

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Example (continued)

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- Now once  $P_i$  `pick_up_forks` starts execution,  $P_i$  will acquire a lock. So, even in the case of context switching its neighbours simply can't acquire lock in `pick_up_forks` until  $P_i$  completes `pick_up_forks`

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- Intuitively, we can be sure that the state changes only when both forks are available. Hence, multiple threads can't access same fork at a time.



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- So, we have recursively shown that all threads will finish.



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